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The fibre bundle approach has been applied to the unified description of the magnetic symmetries of different magnetic structures. In the particular case of the spin glass state the global magnetic coupling constant has been interpreted as a section of the corresponding fibre bundle. The fibre of this bundle makes the space of the Gaussian distributions. Thus one can say that the randomness of the spatial distribution of the magnetic moments in the spin glass state is of the Gaussian-like character.

Keywords Fibre bundle approach; Gaussian distribution; global magnetic coupling constant; Lyapunov theorem; spin glass state; stability magnetic field

Introduction

As the fibre bundle approach will be applied here to the description of the symmetry of magnetic structures let us recall a few definitions. The fibre bundle and its topological structure presents a kind of generalization of the Cartesian product of two spaces with arbitrary dimensions.

A fibre bundle consists of [1]:

a. E – total space,
b. M – base manifold,
c. $\mathbf{E}$ – fibre (a kind of space, which is ascribed to every point of M and is "parametrized" by this point),
d. G – a structural group which acts in the standard fibre $\mathbf{E}$,
e. projection π: $E \rightarrow M$.

Global Magnetic Coupling Constant

Spin glass state model for the case of random distribution of dopants and defects below the percolation threshold in the ferromagnetic matrix has been proposed by the authors [2]. In this case the Hamiltonian assumes the following form:

$$\widehat{H} = -\sum_{j \neq i} \sum_{i=1}^{N_j - n_j} J_{ij} \widehat{S}_i \cdot \widehat{S}_j - \sum_{j \neq i} \sum_{i=1}^{n_j} J'_{ij} p(i,\ j) \widehat{S}_i \cdot \widehat{S}_j, \tag{1}$$

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where N_j is a number of magnetic ions surrounding a jth magnetic ion and interacting with it with a coupling constant J_{ij}, n_j is a number of dopants (or defects) surrounding a jth ion, J'_{ij}-coupling constant between ith and jth ion in the presence of a dopant (or a defect), $p(i, j)$ is probability of appearance of a dopant (or a defect) between ith and jth ions [2].

A quantum-mechanical equation for magnetization **M** has been derived and solved after linearization [2]. This equation contains a. o. the following expression:

$$\widetilde{J}(-\mathbf{q}') = \sum_{j\neq i} \sum_{i=1}^{N_j - n_j} J_{ij} e^{i\mathbf{q}'(\mathbf{R}_i - \mathbf{R}_j)} + \sum_{j\neq i} \sum_{i=1}^{n_j} J'_{ij} p(i,j) e^{i\mathbf{q}'(\mathbf{R}_i - \mathbf{R}_j)} \quad (2)$$

where $\mathbf{R}_i$, $\mathbf{R}_j$ – the radius-vectors of the ith and jth magnetic ions, respectively.

Note that the random distribution of either dopants or defects $p(i, j)$ leads to the random distribution of the magnetic coupling constants $p(J'_{ij})$ and – referring to the Anderson spin glass state model (e.g. [3]) – one comes to the conclusion that the latter effect causes the frustration of the magnetic couplings and the appearance of the spin-glass state in the ferromagnetic system under consideration containing either dopants or defects.

The above reasoning is confirmed by Eq. (2), because it expresses – as a Fourier transform – a kind of a "global" magnetic coupling constant $\widetilde{J}(-\mathbf{q}')$, which turns out to be random, the latter characteristic following at least from the dependence of the second term of Eq. (2) on $p(i, j)$. Moreover, the randomness of $\widetilde{J}(-\mathbf{q}')$ seems to be of a Gaussian type, because the random quantity $\widetilde{J}(-\mathbf{q}')$ is expressed in Eq. (2) as a sum of many different random quantities, the latter effect making typical conditions of the functioning of the central limit theorem of the theory of probability, i.e., the Lyapunov theorem. As it follows from the linearized quantum-mechanical equation for magnetization **M** mentioned above and from Eq. (2), also the magnetization in the spin-glass state becomes a random quantity, which obtains the Gaussian-like distribution, too (the Lyapunov theorem again applies) [2]. In other words introducing either defects or dopants changes the relation of strengths of all the coupling constants acting between the magnetic atoms. In the case of the equalization of strengths of all the different types of the coupling constants acting in the crystal a frustration of the spin orientation appears bringing to the spin glass states. Very often the energetic equilibrium of the spin glass state consists in the creation of the ferromagnetic and antiferromagnetic clusters in the crystal. On the other hand the electrical conductance in the presence of the external magnetic induction is related to the reconstruction of these clusters in two cases, namely: (i) if with the increase of the external magnetic induction the volume of the ferromagnetic clusters increases at the cost of the antiferromagnetic ones, then one deals with the appearance of the negative giant magnetoresistance, (ii) in the reverse situation the positive giant magnetoresistance appears [4,5]. Note that ascribing a representative magnetization vector to every cluster one can say – according to what was said above – that the orientation of these magnetization vectors undergoes the Gaussian statistics, too. This effect is accompanied by the significant changes of conductance in the sample leading to either the negative or the positive giant magnetoresistance. The mechanism of these changes concerns the conductance bridges whose number rises in the ferromagnetic clusters and declines in the antiferromagnetic ones, both in the case of the volume increase of the ferromagnetic clusters and the volume decrease of the antiferromagnetic ones. In the reverse situation the insulating bridges tend to dominate. It is obvious that the statistics of both these kinds of bridges makes the replica of the statistics of the orientation of the magnetization vectors mentioned above.

Global coupling constant $\widetilde{J}(-\mathbf{q}')$ can be interpreted as a section of the fibre bundle E, which consists of a base space (being here the momentum space $\mathbf{q}$) and a fibre (being a space built of the Gaussian distributions). Thus the fibre has here two dimensions: σ – variance and μ – mean value. These two values depend on a given point of the momentum space $\mathbf{q}$.

Note that a Fourier transformation of the global coupling constant $\widetilde{J}(-\mathbf{q}')$ from the momentum space $\mathbf{q}$ to R^3 space transforms also its properties related to the central limit theorem of the theory of probability, i.e., to the Lyapunov theorem. It allows us to conclude explicitly that once a Gaussian distribution of the global magnetic coupling constants exists in the real space, then a certain magnetic field should also exist to make possible to create this distribution or – in other words – to create a Gaussian distribution of the magnetic moments of the separate magnetic atoms, the latter distribution making the "magnetic structure" of a spin glass state.

Magnetic Structures and Their Symmetries

To describe the symmetry of the magnetic structures one needs to formulate the corresponding magnetic symmetry groups whose action on these structures leaves them invariant. Several attempts in this respect have been undertaken, e.g., spin groups [6–8], generalized color groups [9–11] and wreath groups [12–14]. The extension of the spin groups to the description of quasicrystals is presented in [15].

To describe the symmetry of a magnetic structure in terms of the fibre bundles one needs a 6-dimensional space E_6. This space is a vector bundle and is represented locally as a Cartesian product of R^3 and a certain vector space V_3 which is spanned by the orthogonal vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$. The vector space V_3 is the fibre of E_6. Note that the crystal structure itself can be fully described in R^3 whereas the spin structure can be fully described in V_3.

A magnetic structure in V_3 is represented by a pair ($\mathbf{M}$, p) where $\mathbf{M}$ is here a kind of magnetization vector in V_3 and p is a point in R^3 of the attaching of the vector $\mathbf{M}$ on a given crystal plane. Thus $\mathbf{M}$ represents the resulting magnetic moment of this plane. The position vector of the point p has coordinates: (0, 0, na/l), where $n = 0, \pm1, \pm2, \pm3, \ldots$, l is the distance between the magnetic lattice planes and a is a certain scaling factor.

The vector $\mathbf{M}$ after a certain modification (see below Section 4) can be interpreted as a section of the vector bundle E_6. This section should have a special form because in the physical system the magnetic moments are localized on the magnetic atoms and vanish outside these atoms in the ideal crystal. The types of these sections are given in [16] for a variety of magnetic structures.

1. The procedure of bringing to continuity the vectors $\mathbf{M}$

To make possible the use of the fibre bundles approach one has to assure that these sections (vectors $\mathbf{M}$) be continuous [16]. Let us modify first a magnetic moment $\mathbf{s_i}$ of the ith magnetic atom in the following way:

$$\mathbf{s}(\mathbf{r}) = \mathbf{s_i} \exp\left[-\frac{(\mathbf{r} - \mathbf{r_i})^2}{a^2}\right] \tag{3}$$

where $\mathbf{r}_i$ is the coordinate of the ith magnetic atom, a is the characteristic scale of the atomic sizes. Such a situation seems to be more physical, because – as a matter of fact – a magnetic moment presents a field which is significant only close to the crystal positions of the magnetic atoms and decreases outside very quickly. The vector **M** itself presents a vector sum of the atomic magnetic moments on a crystal plane. Therefore according to the central theorem of the theory of probability (the Lyapunov theorem) the analogous formula to the above equation is valid for the vector **M**:

$$\mathbf{M_m}(\mathbf{r}) = \mathbf{M_m} \exp\left[-\frac{(\mathbf{r}-\mathbf{r}_l)^2}{d^2}\right] \tag{4}$$

where $\mathbf{r}_l$ is the coordinate of the lth crystal plane, d is a distance between the neighbour crystal planes. The index m corresponds to the appropriate structures: m = f, a, s, fs, ss, t, l, denoting ferromagnetic structure, antiferromagnetic structure, simple spiral, ferromagnetic spiral, skew spiral, transverse spin wave, longitudinal spin wave, respectively. The exponential factor guarantees here that the section **M** is continuous and has small values outside the crystal plane. Note that the Gaussian factor introduced here plays a double role: it makes the vector **M** to be a field and simultaneously makes the description of the magnetic structures more physical. In this paper a description is proposed of the magnetic structures in terms of the fibre bundles [16]. Such an approach turns out to be the most general, because it is based on the most general product of two arbitrary spaces, namely the Cartesian product, which is very suitable to the combination of two "worlds", e.g., the "world of positions" (R^3) and the "world of spins" (V_3). Thus – as it was mentioned above – the description of crystal structures is to be carried out in R^3, whereas the description of spin structures is to be carried out in V_3. The total magnetic symmetry group G_m in the space E_6 is the tensor product of the magnetic symmetry group G_m and the space symmetry group G_Λ of the crystal structure, where m = f, a, s, fs, ss, t, l.

The explicit formulas of the total magnetic symmetry groups for these seven magnetic structures have been derived by the authors in [16].

Discussion

The theory of the authors describing the appearance of the spin glass state has been elaborated based on the assumption of the random distribution of both dopants and defects below the percolation threshold [2].

The authors have also proved that a certain minimum magnetic field is necessary to appear spontaneously for the stability of the spin glass state [17]. They have determined the value of this minimum magnetic field for some spinel families under study [17]. As it was shown above, it turns out that in the spin glass state the magnetic moments have the random Gaussian-like spatial distribution. It seems that the only cause of such a distribution could be here a certain magnetic field arising probably because of the randomness of the distribution $p(i, j)$ of dopants and defects. In the present paper an attempt is also undertaken to explain the relation between the Gaussian-like distribution mentioned above and the magnetic field generating it from one side and this minimum magnetic field which stabilizes the spin glass state from the other side. It is clear that the magnetic field generating the spatial

Gaussian-like distribution of the magnetic moments in the spin glass state can assume the values in the range from this minimum stabilizing magnetic field up to a certain value of the magnetic field which would lead to the ferromagnetic state.

The fibre bundle approach equates the symmetry analysis of magnetic structures with the method of the higher dimensional embeddings of the modulated structures. The symmetry groups appearing in the symmetry analysis become structural groups G of the bundles. From the other side the higher dimensional space needed to the description of a modulated structure makes here the total space of the bundle. One can then say that these three methods, namely the symmetry analysis, the higher dimensional embeddings and the fibre bundle approach are equivalent. In other words the fibre bundle approach can be applied to the description of all the other aperiodic structures, like e.g., the modulated nonmagnetic structures, quasicrystals (nonmagnetic and magnetic) etc. Note that the wreath groups concept introduced by Litvin [12–14] resembles to some extent the fibre bundle approach because a wreath group acts in a similar way as a structural group of the fibre bundle.

It is worthwhile to mention here that these different magnetic structures have been found by the authors to be related with the values of certain topological invariants [18].

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